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LATERAL STABILITY OF AN AIRCRAFT AND VIBRATIONS OF THE AILERONS--ETC(U)  
JUN 77 J MARYNIAK, M ZLOCKA  
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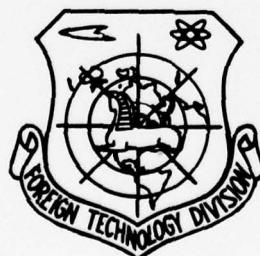
## FOREIGN TECHNOLOGY DIVISION



LATERAL STABILITY OF AN AIRCRAFT AND VIBRATIONS  
OF THE AILERONS WHILE TAKING INTO CONSIDERATION  
ELASTIC DEFORMATION OF THE WINGS AND THE  
ELASTICITY OF THE CONTROL SYSTEM

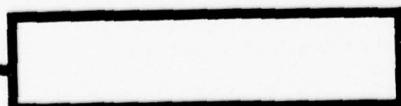
by

J. Maryniak, M. Zlocka



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LATERAL STABILITY OF AN AIRCRAFT AND VIBRATIONS OF THE AILERONS WHILE  
TAKING INTO CONSIDERATION ELASTIC DEFORMATION OF THE WINGS AND THE  
ELASTICITY OF THE CONTROL SYSTEM

BY Jerzy Mariniak, and Maria Złocka ((Warsaw)

1. Introduction

In this work there has been investigated the influence of stiffness and damping in the system of control (guidance) by ailerons while taking into consideration the elastic deformation of the wings on the lateral stability of the aircraft as well as the vibrations of the ailerons. The aircraft has been treated as a rigid mechanical system with elastically deformable wings and movable ailerons.

The equations of motion have been worked out in quasi-coordinates using the Boltzmann-Hamel equations /4/ for mechanical systems with holonomic constraints in a system of coordinates connected with the aircraft.

In the work the assumption has been made that the aerodynamic forces and moments do not have any influence on the form and frequency of the free vibrations of the wings. This assumption has made possible a special examination of each inherent form of wing vibrations. The wings constituting a continuous system of an infinite number of degrees of freedom have been substituted for by a precisely determined number of degrees corresponding to the number of assumed forms. The forms and frequencies of the free vibrations have been determined

experimentally by way of resonance investigations./6,8/.

The linearization of the equations of motion was carried out on the basis of the theory of small perturbations /1,2,3,7,20/. It was assumed that antisymmetric motions of the aircraft cause exclusively antisymmetric changes of the aerodynamic forces and moments, moreover symmetric---symmetric changes of the aerodynamic loads. The above assumptions made possible the resolution of the system of equations /1,2,3,20/, which describe an arbitrary motion of the aircraft, into two systems: the system of equations of symmetric motions /8,9,13/ of longitudinal stability, and the system of equations of <sup>antisymmetric</sup> <sub>Λ</sub> motions of lateral stability /14/.

There have been taken into account five degrees of freedom, of these three are the degrees of freedom of the rigid aircraft: roll  $\phi$ , yaw  $\psi$ , the velocity of lateral displacements  $v$  as well as the anti-symmetric elastic deformations of the wings  $\zeta$  and the elastic deflection of the ailerons  $\beta$  /14/.

After linearization of the system of equations, the solution was carried out as far as the designating of the eigenvectors and the eigenvalues of the matrix of state corresponding to them. Sample numerical computations have been carried out for an airplane of the "Wilga" class in accordance with their own programs in the Establishment for Numerical Calculations of the University of Warsaw.

In the accessible literature having to do with the dynamics of moving objects, one does not meet with a derivation of the equations

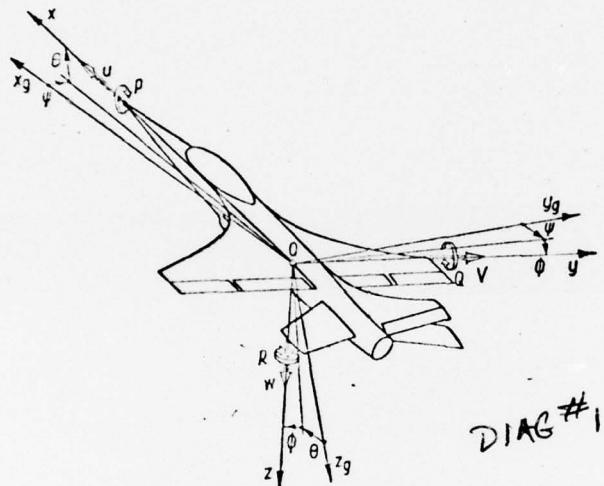
of motion thru use of the Hamel-Bolzmann equations for systems of holonomic constraints. Properly, the use of the Boltzmann-Hamel equations /4/ for the deriving of the equations of motion of mobile (moving) objects in a system of coordinates connected to that object makes possible in a relatively simple manner a taking into consideration of the degrees of freedom that result from the deformability of a body as also the relative motions of the elements of the object being examined /9,13,14/.

The Boltzmann-Hamel equations are generalized Lagrange equations of the second type expressed in quasi-coordinates and quasi-velocities. The quasi-velocities are linear combinations of the generalized velocities whose coefficients are dependent on the generalized coordinates /4,19/, where one can include also free (arbitrary) terms as also clearly (those) dependent on time. /19/. In the case considered such quasi-velocities are kinematic parameters of motion determined in a system of central axes of reference, rigidly connected with the aircraft. The kinematic parameters mentioned above are the angular velocities of the aircraft  $P, Q, R$  as well as the linear velocities of its center of mass  $U, V, W$  /1,2,3,4,9,13,14,19, and 20/.

The equations of motion introduced in the third section of the present work are universal and one can apply them directly to the description of the motion of arbitrary deformable <sup>mobile</sup> objects in the assumed system of reference.

## 2. The Assumed System of Reference

For the description of the dynamics of an aircraft three systems of reference are indispensable: the gravitational system precisely connected with the earth  $Ox_1y_1z_1$ , the velocity system connected with the flow  $Ox_{a_1}y_{a_1}z_{a_1}$ , as well as the one connected rigidly with the aircraft  $Oxyz$ .



Diag #1 The assumed system of reference  $Oxyz$  connected with the aircraft as well as the introduced linear and angular velocities.

The momentary situation of the aircraft as a rigid body is determined by the situation by the center of mass of the object  $\bar{r}_i(r_1, r_2, r_3)$ , measured with respect to the immovable system of coordinates  $Ox_1y_1z_1$  connected with the earth as well as of the angles of rotation of the aircraft  $\psi, \theta, \phi$ .

The angles of rotation determine uniquely the situation of the system of coordinates precisely connected with the aircraft  $Oxyz$  with respect to the gravitational system of coordinates  $Ox_gy_gz_g$  which is parallel to the immovable system  $Ox_1y_1z_1$  (diag #1).

The assumed angles of rotation are quasi-Euler angles, called also aircraft (angles) [1,2,19].

The names of these angles are as follows:  $\phi$  - the angle of roll,  $\theta$  - the angle of pitch,  $\psi$  - the angle of yaw.

The motion of an aircraft was described in the central system  $Oxyz$  rigidly connected with the aircraft, around axes directed as on diag #1 and diag #2.

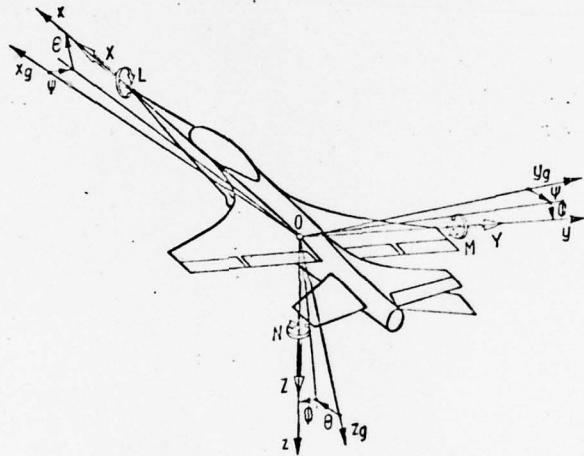


Diagram #2 Assumed components of forces and moments in the reference system  $Oxyz$  connected with the aircraft.

The components of the momentary velocity vectors, linear  $\vec{V}_c$  and angular  $\vec{\Omega}$  in the assumed coordinate system ((diag #1) are as follows:  
--- vector of linear velocity  $\vec{V}_c$

$$(1) \quad \vec{V}_c = U\vec{i} + V\vec{j} + W\vec{k},$$

where  $U$  designates the longitudinal velocity,  $V$  the lateral velocity and  $W$  - the velocity of vertical (plumb) displacements.

-- the vector of momentary angular velocity  $\vec{\Omega}$

$$(2) \quad \vec{\Omega} = P\vec{i} + Q\vec{j} + R\vec{k},$$

where  $P$  is the angular velocity of roll,  $Q$ --the angular velocity of pitch,  $R$ --the angular velocity of yaw.

The vectors of the external forces and the moments of external forces acting on the aircraft have the form (diag #2):

---vector of the external forces  $\vec{F}$

$$(3) \quad \vec{F} = X\vec{i} + Y\vec{j} + Z\vec{k},$$

where  $X$  signifies the longitudinal force,  $Y$ --the lateral force,  $Z$ --the vertical force

---the vector of the main (overall) moment  $\vec{M}$

$$(4) \quad \vec{M} = L\vec{i} + M\vec{j} + N\vec{k},$$

where  $L$  is the roll moment,  $M$ --the pitch moment, and  $N$ --the yaw moment.

The angular velocities  $P, Q, R$  are linear combinations of the generalized velocities  $\dot{\phi}, \dot{\theta}$  and  $\dot{\psi}$  with coefficients dependent upon the generalized coordinates  $\phi, \theta$  and  $\psi$  and are expressed in the following form:

$$(5) \quad \begin{bmatrix} P \\ Q \\ R \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \Lambda_2 \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}.$$

The kinematic relations between the linear velocities  $\dot{x}_1, \dot{y}_1, \dot{z}_1$  measured in the non-moving system  $Ox_1y_1z_1$  and the components of velocity  $U, V, W$  are the following:

$$\begin{aligned}
 (6) \quad & \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \Lambda_V \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dy_1}{dt} \\ \frac{dz_1}{dt} \end{bmatrix}, \\
 & \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} \cos \Theta \cos \Psi & \cos \Theta \sin \Psi & -\sin \Theta \\ \sin \Phi \sin \Theta \cos \Psi & -\sin \Phi \sin \Theta \sin \Psi & \sin \Phi \cos \Theta \\ -\cos \Phi \sin \Psi & \cos \Phi \cos \Psi & \cos \Phi \cos \Theta \end{bmatrix} \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dy_1}{dt} \\ \frac{dz_1}{dt} \end{bmatrix}.
 \end{aligned}$$

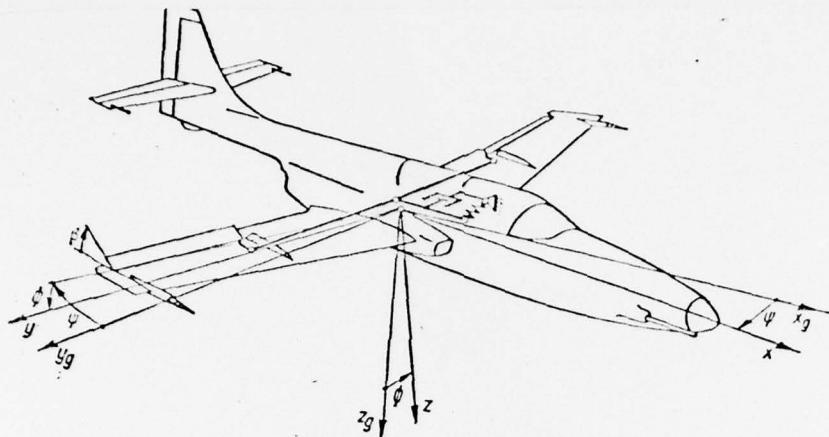
The relations (5) and (6) determine the kinematic parameters that are quasi-velocities.

### 3. Equations of Motion of a Deformable Free Object

The model of a non-deformable aircraft, most frequently encountered in the litterature, cannot always be assumed (accepted) in an investigation of the dynamic properties of an object. Some rigid and elastic relative motions can have a substantial influence on the character of the motion of an aircraft. In the case of taking into consideration the elastic flexibility of the wings, one obtains a system of an infinite number of degrees of freedom of motion. Practical carrying out of the computations for such a system is impossible and, therefore, there is also used an approximate method. It is based on the assumption that the aerodynamic forces and moments do not change the non-elastic free vibrations of the wings.

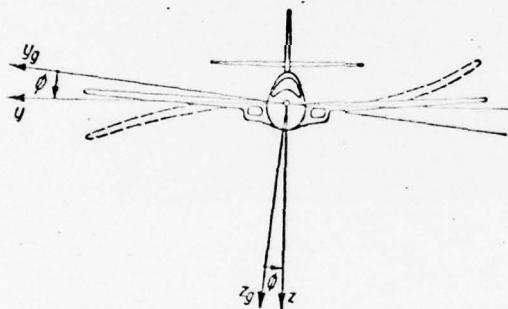
The sag of a wing (diag #4) is described by the function:

$$z(y, t) = \sum_{i=1}^n h_i(y) \zeta_i(t),$$



Rys. 3. Przyjęty model samolotu i przemieszczeń kątowych obrotów antysymetrycznych samolotu

Diag #3: Assumed model of the aircraft and of the angular displacements of the antisymmetric rotations of the aircraft.



Rys. 4. Przyjęty model ruchów przechylających i odkształceń giętnych skrzydeł samolotu

Diag #4: Assumed model of the rolling motions and elastic deformations of the wings of the aircraft.

where  $h_i(y)$  is the successive form of the free vibrations. This has made possible an examination of the influence on the motion of the aircraft of each form of the vibrations separately. In agreement (conformity) with the above that correspond to the  $i$ th form have been presented as follows:

$$(7) \quad \xi_i(y, t) = h_i(y) \zeta_i(t).$$

There has been taken into consideration also the motion of the ailerons, which is possible in spite of the blocked control stick, due to the existence of elastic deformations in the system of control by ailerons. The displacement of the ailerons is determined by the angle of rotation of the aileron  $\varphi$  around the axis of the hinges (diag 3)

The equation of motion of the aircraft has been introduced in quasi-coordinates, using the Boltzmann-Hamel equations for holonomic systems /4/. The Boltzmann-Hamel equations are generalized Lagrange equations of the second type for non-inertial systems described in quasi-coordinates, and they have the following form:

$$(8) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial \omega_\mu} \right) - \frac{\partial T^*}{\partial \pi_\mu} + \sum_{r=1}^k \sum_{z=1}^k \gamma_{z\mu} \frac{\partial T^*}{\partial \omega_r} \omega_z = Q_\mu^*,$$

where  $\mu, r, z = 1, 2, \dots, k, k$  signifies the number of degrees of freedom,  $\omega_\mu$  -- the quasi-velocities,  $T^*$  -- the kinetic energy in quasi-velocities,  $\pi_\mu$  -- the quasi-coordinates,  $Q_\mu^*$  -- the generalized forces.

The relations between the quasi-velocities and the generalized velocities has the form:

$$(9) \quad \omega_\sigma = \sum_{z=1}^k a_{\sigma z} \dot{q}_z,$$

$$(10) \quad \dot{q}_\sigma = \sum_{\mu=1}^k b_{\sigma\mu} \omega_\mu,$$

where  $\dot{q}_\sigma$  signifies the generalized velocities,  $a_{\sigma z} = a_{\sigma z}(q_1, q_2, \dots, q_k)$ ,  $q_k$  -- the generalized coordinates,  $b_{\sigma\mu} = b_{\sigma\mu}(q_1, q_2, \dots, q_k)$ , whereby there exists the following matrix dependence:

$$(11) \quad [a_{\sigma z}] = [b_{\sigma\mu}]^{-1}.$$

The triple indexed multipliers (coefficients) of Boltzmann are determined by the relationship:

$$(12) \quad \gamma_{\mu}^{\nu} = \sum_{\sigma=1}^k \sum_{\lambda=1}^k \left( \frac{\partial a_{\sigma\lambda}}{\partial q_{\mu}} - \frac{\partial a_{\mu\lambda}}{\partial q_{\sigma}} \right) b_{\sigma\lambda} b_{\nu\mu}.$$

In the case when the quasi-coordinates are generalized coordinates, then the triple-indexed Boltzmann coefficients  $\gamma_{\mu}^{\nu}$  (12) are equal to zero.

In the assumed model of a deformable aircraft the vector of the quasi-velocities is the following:

$$(13) \quad \omega = \text{col}[U, V, W, P, Q, R, \dot{\beta}, \dot{\zeta}],$$

where  $U, V, W, P, Q, R$  determine the relations (5) and (6), and the vector of quasi-coordinates corresponding to it has the form:

$$(14) \quad \pi = \text{col}[\pi_U, \pi_V, \pi_W, \pi_P, \pi_Q, \pi_R, \beta, \zeta].$$

The vector of the generalized coordinates is as follows:

$$(15) \quad q = \text{col}[x_1, y_1, z_1, \phi, \theta, \psi, \beta, \zeta].$$

The matrix  $[a_{\mu\nu}]$  in the case of the assumed model (diag#3 and diag #4) in the assumed system of coordinates (diag#1) in conformance with the relations (5) and (6) has the following form:

$$(16) \quad [a_{\mu\nu}] = \begin{bmatrix} \Lambda_V & 0 & 0 \\ 0 & \Lambda_Q & 0 \\ 0 & 0 & I \end{bmatrix},$$

whereby the matrix corresponding to it  $[b_{\mu\nu}]$  is determined in the form:

$$(17) \quad [b_{\mu\nu}] = [a_{\mu\nu}]^{-1} = \begin{bmatrix} \Lambda_V^{-1} & 0 & 0 \\ 0 & \Lambda_Q^{-1} & 0 \\ 0 & 0 & I \end{bmatrix}.$$

The majority of the Boltzmann coefficients  $\gamma_{\alpha\beta}$  in the case of the assumed deformable aircraft are equal to zero. One of the non-zero Boltzmann coefficients is  $\gamma_{54}^6$ . It is computed in accordance with the following relationship:

$$(18) \quad \gamma_{54}^6 = \sum_{\alpha=1}^8 \sum_{l=1}^8 \left( \frac{\partial a_{6\alpha}}{\partial q_l} - \frac{\partial a_{6l}}{\partial q_\alpha} \right) b_{l5} b_{54}.$$

The quasi-coordinates  $\beta$  &  $\zeta$  are generalized coordinates. In conformity with the above

$$\frac{\partial a_{6\alpha}}{\partial \beta} = 0; \quad \frac{\partial a_{6l}}{\partial \zeta} = 0$$

for  $\alpha, l = 1, 2, \dots, 8$ , is simplified and the relationship (18) assumes the form

$$\gamma_{54}^6 = \sum_{\alpha=1}^6 \sum_{l=1}^6 \left( \frac{\partial a_{6\alpha}}{\partial q_l} - \frac{\partial a_{6l}}{\partial q_\alpha} \right) b_{l5} b_{54}.$$

Analyzing the matrices  $[a_{\alpha\beta}]'$  and permits us to note that:

$$b_{14} = b_{24} = b_{34} = b_{54} = b_{64} = 0; \quad b_{44} = 1;$$

$$a_{61} = a_{62} = a_{63} = a_{64} = 0; \quad b_{15} = b_{25} = b_{35} = 0;$$

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$$\gamma_{54}^6 = - \frac{\partial a_{65}}{\partial q_4} b_{55} b_{44} - \frac{\partial a_{66}}{\partial q_4} b_{65} b_{44},$$

WHERE  
gdzie

$$\frac{\partial a_{65}}{\partial q_4} = \frac{\partial (-\sin \varphi)}{\partial \varphi} = -\cos \varphi,$$

$$\frac{\partial a_{66}}{\partial q_4} = \frac{\partial (\cos \varphi \cos \Theta)}{\partial \varphi} = -\sin \varphi \cos \Theta,$$

WHEREBY  
przy czym

$$b_{55} = \cos \varphi, \quad b_{65} = \frac{\sin \varphi}{\cos \Theta}.$$

AS A RESULT WE GET

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$$\gamma_{54}^6 = \cos^2 \varphi + \sin^2 \varphi = 1.$$

The remaining non-zero Boltzmann coefficients are computed analogously: they have the following values:

$$\gamma_{46}^5 = 1, \quad \gamma_{64}^5 = -1,$$

$$\gamma_{45}^6 = -1, \quad \gamma_{54}^6 = 1,$$

$$\gamma_{34}^3 = -1, \quad \gamma_{43}^3 = 1.$$

$$\gamma_{26}^1 = -1, \quad \gamma_{62}^1 = 1,$$

$$\gamma_{15}^1 = 1, \quad \gamma_{53}^1 = -1,$$

$$\gamma_{16}^2 = 1, \quad \gamma_{61}^2 = -1,$$

$$\gamma_{15}^3 = -1, \quad \gamma_{51}^3 = 1,$$

$$\gamma_{14}^3 = 1, \quad \gamma_{42}^3 = -1,$$

$$\gamma_{56}^4 = -1, \quad \gamma_{65}^4 = 1,$$

After introducing the thus computed Boltzmann coefficients into the equation (8) we get equations of motion for an arbitrary free object whose motion is described in the assumed system of reference. The equations of motion take on the following form:

$$(19) \quad i \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial U} \right) - \frac{\partial T^*}{\partial \pi_U} - \frac{\partial T^*}{\partial V} R + \frac{\partial T^*}{\partial W} Q = Q_U^*,$$

$$(20) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial V} \right) - \frac{\partial T^*}{\partial \pi_V} + \frac{\partial T^*}{\partial U} R - \frac{\partial T^*}{\partial W} P = Q_V^*,$$

$$(21) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial W} \right) - \frac{\partial T^*}{\partial \pi_W} - \frac{\partial T^*}{\partial U} Q + \frac{\partial T^*}{\partial V} P = Q_W^*,$$

$$(22) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial P} \right) - \frac{\partial T^*}{\partial \pi_P} - \frac{\partial T^*}{\partial V} W + \frac{\partial T^*}{\partial W} V - \frac{\partial T^*}{\partial Q} R + \frac{\partial T^*}{\partial R} Q = Q_P^*,$$

$$(23) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial Q} \right) - \frac{\partial T^*}{\partial \pi_Q} + \frac{\partial T^*}{\partial U} W - \frac{\partial T^*}{\partial W} U + \frac{\partial T^*}{\partial P} R - \frac{\partial T^*}{\partial R} P = Q_Q^*,$$

$$(24) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial R} \right) - \frac{\partial T^*}{\partial \pi_R} - \frac{\partial T^*}{\partial U} V + \frac{\partial T^*}{\partial V} U - \frac{\partial T^*}{\partial P} Q + \frac{\partial T^*}{\partial Q} P = Q_R^*,$$

$$(25) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial \beta} \right) - \frac{\partial T^*}{\partial \beta} = Q_\beta^*,$$

$$(26) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial \xi} \right) - \frac{\partial T^*}{\partial \xi} = Q_\xi^*.$$

The equations (19) & (24) describe the motion of an arbitrary rigid body in a central system of coordinates connected with the object. The remaining two equations are the result of taking into consideration additional degrees of freedom; of the relative motions of the ailerons (25) and the deformation of the wings (26).

In an arbitrary motion of an arbitrary deformable object, the number of equations of the type (25) can be arbitrary and depends exclusively on the number of additionally taken into consideration degrees of freedom in the presence of the unchanging form of the first six equations (19)-(24).

#### 4. Equations of the Antisymmetric Motions of a Deformable Aircraft

In an arbitrary motion of an object, equations (19) thru (26) in general do not resolve themselves into equations describing symmetric and antisymmetric motions. They are very non-linear ordinary differential equations of the second order. The resolving of the equations is only possible in use for studies of the theory of small perturbations with regard to an established motion and linearization of the equations.

In the present work we have assumed that the aircraft carries out only antisymmetric motions (diag 3 and diag #4), that means the yawing motion  $\nu$ , the rolling motion  $\varphi$ , the lateral displacement  $\pi_r$ , the antisymmetric displacements of the ailerons  $\beta$  and the antisymmetric vibrations of the wings  $\zeta$ . In the presence of the above assumptions, one gets a system of five equations in the general form:

$$(27) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial V} \right) - \frac{\partial T^*}{\partial \pi_V} + \frac{\partial T^*}{\partial U} R - \frac{\partial T^*}{\partial W} P = Q_V^*,$$

$$(28) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial P} \right) - \frac{\partial T^*}{\partial \pi_P} - \frac{\partial T^*}{\partial V} W + \frac{\partial T^*}{\partial W} V - \frac{\partial T^*}{\partial Q} R + \frac{\partial T^*}{\partial R} Q = Q_P^*,$$

$$(29) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial R} \right) - \frac{\partial T^*}{\partial \pi_R} - \frac{\partial T^*}{\partial U} V + \frac{\partial T^*}{\partial V} U - \frac{\partial T^*}{\partial P} Q + \frac{\partial T^*}{\partial Q} P = Q_R^*,$$

$$(30) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial \beta} \right) - \frac{\partial T^*}{\partial \beta} = Q_\beta^*,$$

$$(31) \quad \frac{d}{dt} \left( \frac{\partial T^*}{\partial \zeta} \right) - \frac{\partial T^*}{\partial \zeta} = Q_\zeta^*.$$

The whole kinetic energy of the aircraft  $T^*$  computed in quasi-velocities has the following form:

$$\begin{aligned}
 T^* = & \frac{1}{2} [M_s(U^2 + V^2 + W^2) + I_q \dot{\beta}^2 + \beta_2 \dot{\zeta}^2 + I_x P^2 + I_y Q^2 + I_z R^2] - I_{xy} PQ + \\
 & + S_x(WP - UR) + S_y(VR - WQ) + S_z(UQ - VP) + \{ (A_3 + B_1^P + B_3^L)W + \\
 & + (A_1 + B_3^P + B_1^L)P - [A_4 + (B_1^P + B_1^L)x_L - B_3^P - B_3^L]Q \} \dot{\zeta} + [(S_q^P - S_q^L)U + \\
 & + (I_{\zeta q}^P - I_{\zeta q}^L)P + (S_q^P - S_q^L)W] \dot{\beta} + (B_3^P - B_3^L) \dot{\zeta} \dot{\beta},
 \end{aligned}
 \tag{32}$$

where

$$\begin{aligned}
 A_1 &= \int_{-b/2}^{b/2} m_s(y)h(y)y dy, & A_2 &= \int_{-b/2}^{b/2} m_s(y)h^2(y) dy, \\
 A_3 &= \int_{-b/2}^{b/2} m_s(y)h(y) dy, & A_4 &= \int_{-b/2}^{b/2} S_s(y)h(y) dy, \\
 B_1 &= \int_{b/2-b_L}^{b/2} m_L(\eta)h(\eta)d\eta, & B_2 &= \int_{b/2-b_L}^{b/2} m_L(\eta)h^2(\eta)d\eta, \\
 B_3 &= \int_{b/2-b_L}^{b/2} S_L(\eta)h(\eta)d\eta, & B_4 &= \int_{b/2-b_L}^{b/2} m_L(\eta)h(\eta)\eta d\eta,
 \end{aligned}$$

where  $M_s$  signifies the mass of the whole aircraft,  $I_x, I_y, I_z, I_{xy}$  — are moments of inertia and the moment of the deviation of the aircraft with respect to the system of reference, Oxyz:  $S_x, S_y, S_z$  — are the static moments of the aircraft with respect to the reference system Oxyz:  $I_q, I_{\zeta q}, I_{\zeta q}, S_q, S_{\zeta q}$  — are the moments of inertia, deviation and static of the ailerons with respect to the axis of the hinges  $\eta$  and of the axis of symmetry of the aircraft  $\zeta$ ; the upper indices of L & P determine respectively the left and right aileron;  $m_s(y), m_L(\eta)$  — are the distributions of the masses of the wings and of the aileron as a function of span;  $h(y)$  — is the function of the sag of the wing corresponding to the form being investigated of the free vibrations.

The generalized forces appearing on the right sides of the equations (27) thru (31) determine, taking into consideration the potential energy of the deformations of the wings and of the control sys-

system [8, 14], the gravitational forces [1, 2, 3, 8, 14] as well as the aerodynamic forces and moments [1, 2, 3, 8, 14, 20].

The potential energy of the elastic deformations of the wings and of the control system of the ailerons has the following form:

$$(33) \quad U_s = \frac{1}{2} k_\zeta \zeta^2 + \frac{1}{2} k_\beta \beta^2,$$

where  $k_\zeta$  the elastic rigidity of the wings,  $k_\beta$  the stiffness of the system of control of ailerons, where:

$$(34) \quad k_\zeta = \omega^2 \int_{-b/2}^{b/2} m_s(y) h^2(y) dy$$

is the generalized stiffness of the wings that corresponds to the taken-into-consideration form of the free vibrations, which (stiffness) is described by the function  $h(y)$  of frequency of vibrations  $\omega$ .

The viscous damping in the system of control by ailerons has been taken into account by the introduction of Rayleigh's dissipation function  $U_R$ .

$$(35) \quad U_R = -\frac{1}{2} k_\beta \dot{\beta}^2.$$

The components of the force of gravity in the reference system Oxyz have the form

$$(36) \quad m\bar{g} = \mathbf{A}_g mg,$$

where

$$\mathbf{A}_g = \begin{bmatrix} -\sin\Theta \\ \cos\Theta \sin\Phi \\ \cos\Theta \cos\Phi \end{bmatrix},$$

and in the case under consideration, which takes into consideration exclusively anti-symmetric motions

$$(37) \quad Y_g = mg \cos \theta \sin \phi.$$

The aerodynamic forces and moments acting on the aircraft have been introduced while taking into consideration the stationary aerodynamics. Linearization of the aerodynamic forces and moments has been carried out in accordance with the method of Bryan [1, 2, 3, 20]. This method is based on assumption that the aerodynamic moments and forces are momentary functions of the magnitude of the changes of velocity, linear and angular and of their derivatives. These functions have been developed into a Taylor series with respect to the previously mentioned changed. In these series there have been considered only terms of the first order [1, 2, 3, 20].

In the case examined it has been assumed that the aircraft moves with a steady uniform level motion. One assumes that the steady motion of the aircraft is subjected to small perturbations, that means that:

$$(38) \quad \begin{aligned} \Phi &= \varphi, & P &= p, & U &= U_0 = \text{const}, \\ \Theta &= \theta_0 = \text{const}, & Q &= 0, & V &= v, \\ \Psi &= \psi, & R &= r, & W &= 0. \end{aligned}$$

The generalized forces in the equations (27)-(31) when taking into consideration the above perturbations and the introduction of the relations (37) and (38) have the form:

$$(39) \quad \begin{aligned} Q_v^* &= Y_v v + Y_p p + Y_r r + mg \cos \theta_0, \\ Q_p^* &= L_v v + L_p p + L_r r + L_\beta \dot{\beta} + L_\zeta \dot{\zeta} + L_\rho \beta, \\ Q_r^* &= N_v v + N_p p + N_r r + N_\beta \dot{\beta} + N_\zeta \dot{\zeta} + N_\rho \beta, \\ Q_\beta^* &= R_p p + R_r r + R_\beta \dot{\beta} + R_\zeta \dot{\zeta} + R_\rho \beta, \\ Q_\zeta^* &= E_p p + E_r r + E_\beta \dot{\beta} + E_\zeta \dot{\zeta} + E_\rho \zeta, \end{aligned}$$

where  $Y_v = \frac{\partial Y}{\partial v}$ ,  $Y_p = \frac{\partial Y}{\partial p}$ , ...,  $E_\zeta = \frac{\partial E}{\partial \zeta}$  are according to the names accepted in flight lore, the aerodynamic derivatives

[1, 2, 3, 20].

The aerodynamic derivatives that appear in the relations (38) were introduced in [1, 2, 3, 6, 8, 14, and 20].

The system of equations (27) thru (31), after taking into consideration (28) and (39), are converted into a non-dimensional form dividing the equations of the forces by  $\rho V_c^2 S$ , and the equations of the moments by  $\rho V_c^2 Sb/2$ .

- $\hat{t} = \frac{M_s}{\rho V_c S}$  - the aerodynamic time,
- $\mu = \frac{M_s}{\rho Sb/2}$  - the relative density of the aircraft,
- $j_x = \frac{I_x}{M_s(b/2)^2}$  - the non-dimensional moment of inertia,
- $\bar{t} = \frac{t}{\hat{t}}$  - the non-dimensional time,
- $\bar{v} = \frac{v}{V}$  - non-dimensional linear velocity,
- $\bar{p} = p\hat{t}$  - non-dimensional angular velocity,
- $y_v = \frac{Y_v}{\rho S V_c}$  - non-dimensional derivative of lateral force with respect to the change of the linear velocity of sideslip,
- $y_p = \frac{Y_p}{\rho S V_c b/2}$  - non-dimensional derivative of the lateral force with respect to the change of roll angular velocity,
- $n_v = \frac{N_v}{\rho S V_c b/2}$  - non-dimensional derivative of the yawing moment with respect to the change of the linear velocity of sideslip,
- $I_r = \frac{L_r}{\rho S V_c b/2}$  - non-dimensional derivative of the roll moment with respect to the yaw angular velocity change,

In an analogous manner there have been presented in a non-dimensional form the remaining terms (expressions) of the equations.

The system of equations of motion in non-dimensional form has been obtained in matrix notation in the following form:

(40)

$$A\ddot{x} + B\dot{x} + Cx = 0,$$

where

$$\mathbf{x} = \text{col}[\pi_v, \pi_p, \pi_r, \zeta, \beta],$$

$$\dot{\mathbf{x}} = \text{col}[\dot{\vartheta}, \dot{p}, \dot{r}, \dot{\zeta}, \dot{\beta}],$$

whereby

- A is the matrix of the coefficients of inertia, VIZ

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -j_{xz}/j_x & l_z/j_x & j_{z\eta}/j_x \\ 0 & -j_{xz}/j_x & 1 & 0 & 0 \\ 0 & e_p & 0 & 1 & e_p \\ 0 & j_{z\eta}/j_\eta & 0 & r_z/j_\eta & 1 \end{bmatrix},$$

- B is the matrix of coefficients of damping, VIZ

$$B = \begin{bmatrix} -y_\vartheta & -y_p/\mu & (1-y_r/\mu) & 0 & 0 \\ -\mu l_v/j_x & -l_p/j_x & -l_r/j_x & -l_\zeta/j_x & -l_\beta/j_x \\ -\mu n_v/j_x & -n_p/j_x & -n_r/j_x & -n_\zeta/j_x & -n_\beta/j_x \\ 0 & e_p & e_r & e_\zeta & e_\beta \\ 0 & -r_p/j_\eta & -r_r/j_\eta & -r_\zeta/j_\eta & (k_\beta\mu - r_\beta)/j_\eta \end{bmatrix},$$

- C is the matrix of coefficients of stiffness, VIZ

$$C = \begin{bmatrix} 0 & -y_\vartheta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -l_\beta\mu/j_x \\ 0 & 0 & 0 & 0 & -n_\beta\mu/j_x \\ 0 & 0 & 0 & e_\zeta & e_\beta \\ 0 & 0 & 0 & 0 & (k_\beta\mu - r_\beta)/j_\eta \end{bmatrix}.$$

The matrix equation (40) of second order gets reduced to an equation of the first order in the form:

$$(41) \quad P\dot{q} + Qq = 0, \quad q = \begin{bmatrix} x \\ \dot{x} \end{bmatrix},$$

also

$$\mathbf{P} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{C} & -\mathbf{B} \end{bmatrix}.$$

$$(42) \quad \dot{\mathbf{p}} = \mathbf{R}\mathbf{q},$$

where the matrix of state  $\mathbf{R}$  has the form

$$(43) \quad \mathbf{R} = -\mathbf{P}^{-1}\mathbf{Q} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{A}^{-1}\mathbf{C} & -\mathbf{A}^{-1}\mathbf{B} \end{bmatrix}.$$

The solution has been reduced to the designation (evaluation) of the eigenvectors  $\mathbf{q}_{wj}$  and of the eigenvalues corresponding to  $\lambda_{j,j+1} = \xi_{j,j+1} \pm i\eta_{j,j+1}$  of the matrix of state  $\mathbf{R}$  (43)

The general solution has the form

$$(44) \quad \mathbf{q}(t) = \sum_{j=1}^s C_j \mathbf{q}_{wj} \exp(\lambda_j t),$$

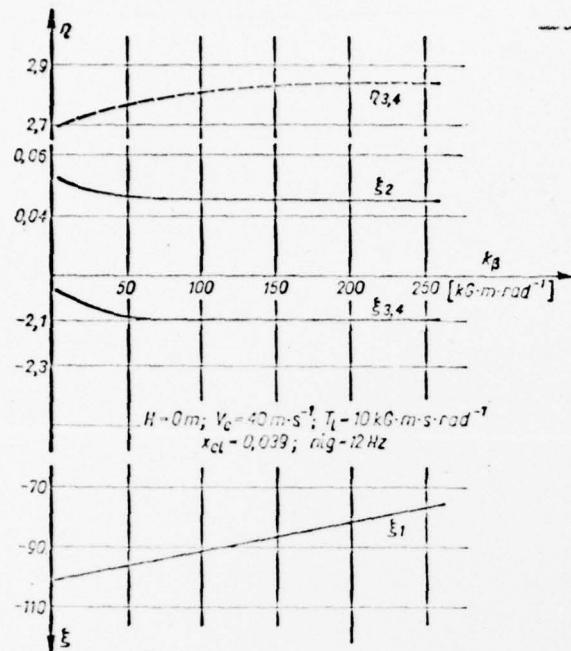
where the  $C_j$  designate constants dependent upon the initial conditions which are the values of the perturbations from steady motion for the moment  $t=0$ ,  $\eta_j$  is the frequency of oscillation of period

$T = \frac{2\pi}{\eta_j} \hat{t}$ ,  $\xi_j$  is the coefficient of damping, whereby  $T_{1/2} = \frac{\ln 2}{\xi_j} \hat{t}$  is the time of damping down of the amplitude to  $\frac{1}{2}$  for  $\xi < 0$ , and in case  $\xi > 0$ , the time of doubling of the amplitude.

## 5. Numerical Example and Suggestions

The example computations have been carried out for a light aircraft of the tourist class "Wilga". We resolve the system of equations (40) evaluating the eigenvectors  $\mathbf{q}_{wj}$  and the corresponding eigenvalues  $\lambda_j$  of the matrix of state  $\mathbf{R}$  (43).

All the computations were carried out in accordance with their own programs at EMC GIER in the Institute of Numerical Calculations of the University of Warsaw.



$\lambda_j$  corresponding to them, that characterize the same motions of the aircraft, wings and ailerons:

$$\lambda_1 = \xi_1$$

characterizes aperiodic displacements of the ailerons  $\beta$  always powerfully damped  $\xi_1 < 0$ ,

$$\lambda_2 = \xi_2$$

characterizes the aperiodic spiral motions that indicate weak instability  $\xi_2 > 0$ ,

$$\lambda_{3,4} = \xi_{3,4} \pm i\eta_{3,4}$$

characterizes periodic oscillations that correspond to rocking turns  $p$  and  $v$  coupled with a yawing motion  $r$  of a motion always damped  $\xi_{3,4} < 0$

$$\lambda_{5,6} = \xi_{5,6} \pm i\eta_{5,6}$$

characterizes motions periodic or aperiodic of the ailerons  $\beta$  coupled with the rolling motions of the aircraft  $p$

$$\lambda_5 = \xi_5$$

damped  $\xi_5 < 0$ , or divergent  $\xi_5 > 0$

$$\lambda_6 = \xi_6$$

$$\lambda_{7,8} = \xi_{7,8} \pm i\eta_{7,8}$$

characterizes elastic vibrations of the wings always damped  $\xi_{7,8} < 0$  of frequency  $\eta_{7,8}$  close to the frequency of the free vibrations of the wings  $\omega$ .

Figure 5. Coefficients of damping  $\xi$  and frequencies of oscillation  $\eta$  of the first four eigenvalues  $\lambda_1$  through  $\lambda_4$  as a function of the stiffness of the system of control of ailerons.

The results have been presented in the form of graphs (Figures 5-9) on which there have been represented by the continuous line changes of the damping coefficients  $\xi_j$  and with a broken line the frequency of oscillation  $\eta_j$ . There have been designated with the same indices on all graphs the eigenvalues correspond-

ing to them, that characterize the same motions of the aircraft, wings and ailerons:

a) Influence of the stiffness of the control system on the stability of the airplane. An increase of stiffness in the control system (Figures 5 and 6) causes a decrease in damping of aperiodic displacements of the ailerons ( $\beta$ )  $\xi_1$  with unchanged characteristics of the spiral motions  $\xi_2$  and of the rocking turn  $\lambda_{34}$  (Figure 5). An increase in the stiffness also does not influence the elastic vibrations of the wings nor the frequency  $\eta_{78}$  as also the damping  $\xi_{78} < 0$ , it has, however, a powerful influence on the damping and character of the motion of the ailerons that is coupled with the roll of the aircraft (Figure 6).

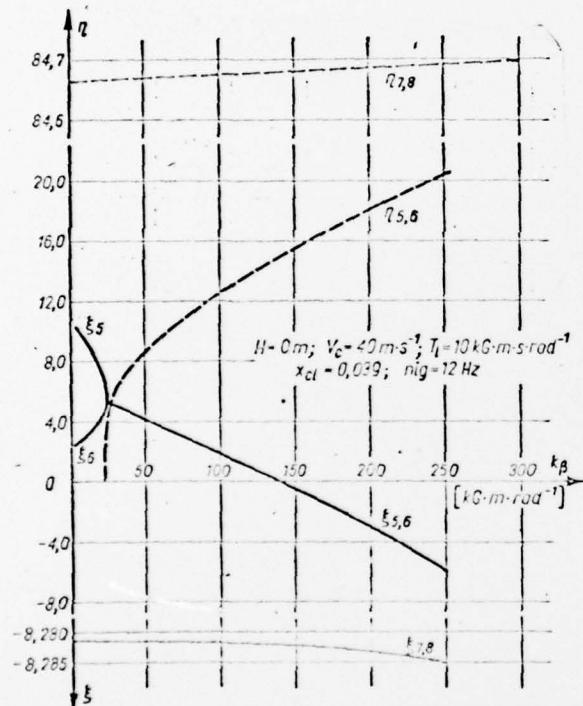


Figure 6. Coefficients of damping  $\xi$  and of frequency of oscillation  $\eta$  of the eigenvalues  $\lambda_5 - \lambda_8$  as a function of the rigidity of the system of control of ailerons.

With small stiffness of the control system of ailerons, the displacements of the ailerons  $\beta$  and the rolling motions of the aircraft  $p$  are aperiodic diverging motions  $\xi_5 > 0$  and  $\xi_6 > 0$ , which in the presence of an increase of stiffness pass over into periodic motions of frequency  $n_{56}$ , initially divergent  $\xi_{56} < 0$ , and then damped  $\xi_{56} < 0$  (Figure 6).

b) Influence of Damping in the System of Control of Ailerons on the Stability of the Aircraft. An increase in the viscous damping in the system of control of ailerons (Figures 7 and 8) causes an increase of the powerful damping  $\xi_1 < 0$  of the aperiodic displacements of the ailerons  $\beta$  (Figure 7) with an unchanged character of the spiral motions of the aircraft  $\xi_2 > 0$  as well as of the frequency  $n_{34}$  and the damping  $\xi_{34} < 0$  of the rocking turns of the aircraft (Figure 7). Damping in the control system also does not affect the frequency  $n_{78}$  and the damping  $\xi_{78} < 0$  of the elastic vibrations of the wings (Figure 8).

A change in the damping in the control system has a decisive and most important effect on the displacement of the ailerons  $\beta$  and coupled with them on the rolling motions of the aircraft  $p$  (Figure 8). In the presence of a small damping there appear harmonic oscillations of frequency  $\xi_{56}$ , initially powerfully divergent  $\xi_{56} > 0$ , passing over into damped  $\xi_{56}$  with a simultaneous decrease in the frequency of oscillation  $n_{56}$ . At a certain critical damping, periodic vibration of the ailerons and wings pass over into powerfully damped aperiodic motions  $\xi_5 < 0$  and  $\xi_6 < 0$  (Figure 8).

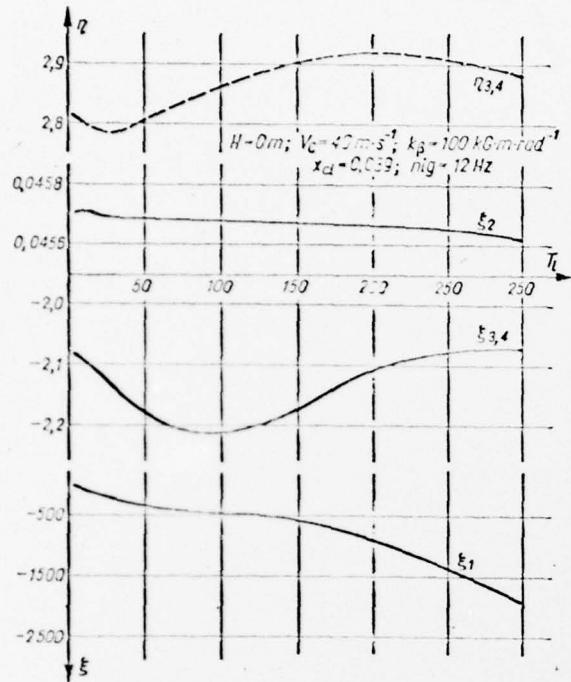


Figure 7.

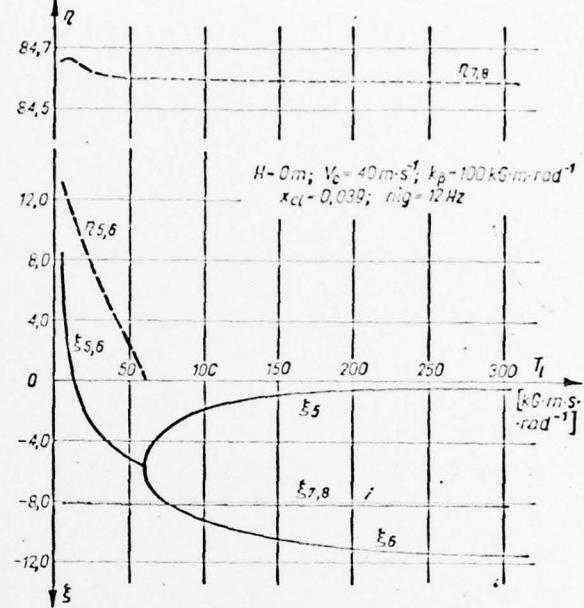


Figure 8.

Figure 7. Damping coefficients  $\xi$  and frequencies of oscillation  $\eta$  of the first four eigenvalues  $\lambda_1 - \lambda_4$  as a function of the damping of the system of control of ailerons.

Figure 8. Coefficients of damping  $\xi$  and frequencies of oscillation  $\eta$  of the eigenvalues  $\lambda_5 - \lambda_8$  as a function of the damping of the system of control with ailerons.

c) Influence of the balance (trim) of the Ailerons on the Stability of the aircraft: Previous balancing of the ailerons effectively influences the dynamic properties of the aircraft as also even the motions of the ailerons themselves by the stabilizing of the aircraft (Figure 9).

A change in the balance has a decisive influence on the displacement of the ailerons  $\beta$  and on the rolling motions of the aircraft p. Zero static balancing just as before (that means the center of mass of the aileron is located on the axis of rotation of the aileron or in front of the axis), causes aperiodic motions powerfully damped  $\beta, \zeta_0$  and  $\zeta_0$ , moreover, rear balancing favors the appearance of periodic oscillations of frequency  $\eta_{K0}$  and of damping  $\zeta_{K0} \zeta_0$ .

#### 6. General Suggestions

Taking into consideration of additional degrees of freedom which are the elastic deformability of the wings and the elastic displacements of the ailerons, in relation to the results obtained in the case of a rigid plane, causes the appearance of four additional eigenvalues.

The eigenvalues  $\lambda_2$  &  $\lambda_{3,4}$  are precise equivalents of the eigenvalues that characterize the motions of a rigid aircraft, that is, spiral motions and rocking turns.

In the case considered, there is a lack of an equivalent that characterizes the aperiodic powerfully damped roll of the rigid aircraft. There arises, moreover, a powerful coupling of the relative displacements of the ailerons  $\beta$  with the rolling motions of the aircraft p.

Powerfully damped elastic vibrations of the wings do not influence

in a substantial manner the remaining motions of the aircraft; they, moreover, exclusively depend on the rigidity of the wings.

The results obtained and the suggestions extracted on the basis of them are fair (proper) for the case considered. The application of them to another type of aircraft or flying object requires additional numerical computations in accordance with the programs worked out.

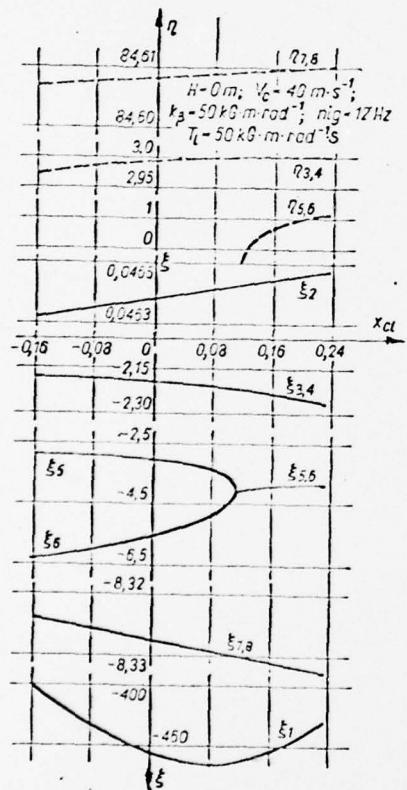


Figure 9. Coefficients of the damping  $\xi$  and of the frequency of oscillation  $\eta$  as a function of the level of static balance of the ailerons.

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Some Abbreviations in the Bibliography

PWN= State Scientific Publishing

WNT= Scientific-<sup>if</sup>Technical Publishing

PW = Warsaw Polytech

Izd Phiz-mat=Physics-Math Publishing

Izd Nauka= Science Publishing

Mashinostroyenie = Mechanical Engineering

Gost Izd Phiz-mat= State Publishing Physics-Mathematics

Mech Teoret i Stos. = Theoretical and Applied Mechanics

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